Study of Hypothetical Learning Trajectories in Mathematics Learning

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ABSTRACT
Mathematics learning in this era requires teachers to be able to develop learning models that pay attention to the characteristics of students, so it is important for teachers to know learning trajectory. This study aims to describe the conception of learning trajectory and hypothetical learning trajectory, as well as presenting the design of hypothetical learning trajectories in mathematics learning. Based on the theories studied, it is concluded that hypothetical learning trajectory is a learning design in the form of allegations on student learning activities based on initial understanding and characteristics of students to achieve higher understanding. HLT has three main components namely learning objectives, learning activities, and hypotheses of the learning process. HLT is used by teachers as a guide to predict and prepare learning flow designs that are appropriate to the stages of student thinking and can improve student learning outcomes. The findings of previous studies show that to design HLT in mathematics learning, educators need to consider student obstacle learning, a hierarchy of material, and learning support capacity, so that learning activities to be hypothesized are built based on these things.

ARTICLE INFO

Key Words:
Hypothetical learning trajectory1; mathematics learning2; learning obstacle3.

INTRODUCTION
Mathematics learning in this era requires active learning to involve students to find knowledge from their thought processes and learning experiences, so students can connect concepts (material) with one another. It is the teacher's job to prepare the right design in the learning process. Isnawan & Wicaksono (2018), explained that educators in applying mathematics learning, do not necessarily without preparation, but must prepare all matters relating to learning. Good learning must have good preparation or design learning processes. Furthermore, Isnawan & Wicaksono (2018) explain a good learning design that is a learning design that is tailored to the needs of students in order to achieve the learning objectives that have been set. The design includes the design of learning objectives, learning strategies, teaching materials used in learning, and learning assessment. Dalziel (2015) also revealed
that the design of learning is a plan related to student activities. These activities must be adjusted to the competencies of the students.

Surya (2018) states that in accordance with the student-oriented learning approach (student center), learning designs designed by teachers need to pay attention to the existence of student learning paths (learning trajectory). Confrey, Gianopulos, McGowan, & Shah (2017), stated that learning trajectory illustrates various ideas that tend to emerge when student-centered learning and a series of tasks that successfully elicit understanding and support cognitive development. Atsnan (2016) states that learning trajectory is a series of activities that children go through in solving a problem or understanding a concept. Learning trajectory provides a plan or pattern that will be used as a reference for making learning plans in each learning process that will be carried out. The use of learning trajectory is expected to be able to develop mathematical thinking competence for students and there is no misunderstanding of concepts. In addition, by developing mathematics learning based on learning trajectory, teachers can learn how to understand students' learning and thinking (Zaman & Hunaifi, 2017).

Suwarto and Purnami (2018), stated that a learning trajectory can never be claimed as the best way to move all students towards understanding. Therefore, learning trajectory is considered a hypothesis even though it has been validated by thousands of students empirically. Nuraida & Arman (2019) also explained that the term learning trajectory (LT) is called hypothetical learning trajectories (HLT) because the design is still in the form of guesses or hypotheses. The term hypothetical learning trajectory (HLT) itself was first put forward and used by Simon (1995) which states that hypothetical learning trajectory consists of three components in the form of learning objectives, learning activities, and the alleged learning process - predictions about how students' thinking and understanding will develop in the context of learning activities. The intended purpose is the achievement of understanding mathematical concepts. The intended learning activity is a series of tasks to find out how students think. The hypothesis of students' way of thinking is intended is the student's thinking flow in understanding the concept of learning (Surya, 2018).

Hypothetical learning trajectory (HLT) which includes a series of instructional assignments in order to understand students towards the concept of mathematics learning, is one of the important aspects that must be possessed by teachers in teaching students meaningful learning. This is because in HLT very consider the knowledge possessed by students. In addition, hypothetical learning trajectory is needed by the teacher to design learning that will be in accordance with students' thinking patterns in class according to student characteristics (Rezky, 2019). Thus, it is important for teachers to know learning trajectory and hypothetical learning trajectory because teachers are expected to be able to develop learning models in schools that pay attention to the characteristics of students based on existing theories and the initial ability of each student so that all students' needs can be fulfilled and also potential students will be more developed with the appropriate learning design.

Therefore, the authors are interested in writing further study of hypothetical learning trajectory in mathematics learning that aims to give readers an overview of hypothetical learning trajectory theoretical concepts and empirical studies of hypothetical learning trajectory that have been developed by previous researchers so that it becomes a description
or design model of hypothetical learning that can be developed continuously to improve the quality of learning.

**METHODOLOGY**

The method used by the author is a literature review method which is carried out by tracing various reference libraries in the form of books, and articles / journals. The library source database used is Google Scholar, DOAJ, and Science Direct. The references used are related to hypothetical learning trajectories and mathematics learning.

The procedure of studying the reference library sources, begins with data collection by collecting references that are relevant to the hypothetical learning trajectories and learning mathematics, then analyzing these findings and presenting further implications of the literature review results in learning activities and further recommendations related to future research related to hypothetical learning trajectories in learning mathematics.

**RESULT AND DISCUSSION**

**Theoretical Results**

**Learning Trajectory**

Learning trajectory is a series of activities that children actually go through in solving a problem or understanding a concept (Nurdin, 2011). Learning trajectory is also defined by Surya (2018) as a flow of students' thinking and understanding abilities that occur in learning activities. Learning trajectory can be built through phenomena that are organized into problem situations where students are immediately involved in the problem being solved (Pamungkas, Yandari & Sukirwan, 2020).

In its implementation, learning trajectory strongly supports the learning process, as explained Nurdin (2011) that a learning path (learning trajectory) provides instructions for teachers to determine and formulate learning goals to be achieved. In addition, Ellis, Ozgur, Kulow, Dogan, & Amidon (2016) suggested that learning trajectories research has the potential to support a better understanding of student learning, enable more effective teaching strategies, and guide better curriculum and standards design. That is, learning trajectory research has the potential to support students' better understanding of learning, enable more effective teaching strategies, and support better curriculum and design standards. So, it can be concluded that learning trajectory is a series of learning activities that are designed with more effective teaching strategies based on problems faced by students to encourage the development of students' thinking to achieve learning goals.

In general the development of children's cognitive abilities starts with concrete things and then gradually leads to abstract things. For each student the journey from concrete to abstract can be different. Some are fast and some are slow. For those who are fast may not require many stages, but for those who are slow will need to go through many stages. Thus for each student may require learning trajectory or a different learning flow.

The difficult thing about determining learning trajectory is the selection of material that must also be adjusted to the development of students. The steps displayed must be adjusted to how their previous learning experience. It is expected that the direction from the teacher must also be flexible and able to adapt to real conditions in the classroom (Zaman & Hunaifi, 2017). So, learning trajectory acts as a guide to the implementation of learning while providing alternative strategies to help students overcome difficulties in understanding the concepts being learned. However, learning trajectory has a weakness in determining the
learning trajectory design that must be adjusted to the initial abilities and development of students, in this case each student has different initial abilities and developments.

In learning mathematics, Zaman & Hunaifi (2017) argues that learning trajectory provides a facility for students to develop mathematical thinking competencies, because it is a tiered trajectory and must be traversed in order to master or move on to the next level.

Suwarto and Purnami (2018), stated that a learning trajectory can never be claimed as the best way to move all students towards understanding. Therefore, learning trajectory is considered a hypothesis even though it has been validated by thousands of students empirically. In addition, Nuraida & Arman (2019) also explained that because the design is still in the form of guesses or hypotheses, the term learning trajectory (LT) is called hypothetical learning trajectories (HLT).

**Definition of Hypothetical Learning Trajectories (HLT).** Arnellis, Suherman, & Amalita (2019), hypothetical learning trajectory is an alleged learning activity that is made as anticipations about what might happen, both the thought process of students who will get learning and things that will occur in the learning process. Furthermore, Rezky (2019) also explains HLT itself creates a hypothesis or teacher's assumptions about how students learn, so the teacher not only considers the material but also sees how the student has understood it or not. Therefore, in the learning process to reach students who learn meaningfully learning designs are needed that are tailored to the characteristics of students.

Confrey, Gianopulos, McGowan, Shah, & Belcher (2017) mention that: "Hypothetical learning trajectories “begin with what students bring to their early understanding of target concepts, and identify landmarks and obstacles students are likely to encounter as they proceed from a naïve to a more sophisticated understanding”" That is, the hypothetical learning trajectory "begins with what students bring to their initial understanding of the concept of purpose, and identifies the most salient things and obstacles that students tend to face as they proceed to higher understanding". So, it can be concluded that hypothetical learning trajectory is a learning design in the form of allegations on student learning activities based on initial understanding and characteristics of students to achieve higher understanding.

**Function of Hypothetical Learning Trajectories (HLT).** Hypothetical learning trajectory will also help teachers to implement models, teaching material strategies and assessments that are appropriate to the stages of student thinking (Surya, 2018). In addition, Rezky (2019) also suggested that learning hypotheses were needed by teachers to design learning that would be in accordance with students’ thinking patterns in class according to student characteristics. Suwarto and Purnami (2018), also suggested that teachers can use HLT to design learning processes that improve student learning outcomes.

Hypothetical learning trajectories (HLT) play a role in research design research, the role and position of HLT at each stage are described by Bakker & Van Eerde (2015) as follows: (1) Preparation and design stage: at this stage, HLT is designed to guide the design process of learning materials to be developed and adjusted, (2) The experimental design stage: during the learning experiment, HLT serves as a guide for what will be the focus of the teacher in the learning process, interviews, and observations. The teacher needs to adjust HLT with learning activities for learning meetings. HLT can change during the teaching experiment phase, (3) Restrospective analysis stage: at this stage HLT acts as a guide in determining the focus of
the analysis, because predictions are made related to student learning processes, can be compared between anticipation of predictions through observation during learning experiments (teaching experiments).

Thus, it was concluded that hypothetical learning trajectory could be used by the teacher/researcher as a guideline to predict and prepare a learning flow design that was appropriate to the stages of student thinking and was expected to improve student learning outcomes.

Component of Hypothetical Learning Trajectories (HLT). Simon (1995), mentions hypothetical learning trajectory includes three components: learning objectives, learning activities, and hypothetical learning processes - a prediction of how students' thinking and understanding will develop in the context of learning activities. Surya (2018) explains more deeply that the intended purpose is the achievement of understanding mathematical concepts. Furthermore, the learning activity in question is a series of assignments to find out how students think. The hypothesis of students’ way of thinking is intended is the student's flow of thought in understanding the concept of learning. Clements & Sarama (2004), mentions a complete hypothetical learning trajectory consisting of three aspects: learning objectives, progressive development, and sequence of task instructions.

The same thing was also expressed by Larson, Wawro, & Zandieh (2017) who explained that in elaborating HLT, HLT was considered as a story line about teaching and learning that occurred over a long period of time. The story line has four interrelated aspects: (1) Learning objectives about student reasoning; (2) The sequence of teaching assignments in which students are involved; (3) Development of students' mathematical activities; (4) The role of the instructor in supporting students' mathematical development throughout the sequence of assignments.

From the three opinions above, it can be concluded that hypothetical learning trajectory in general has three main components consisting of learning objectives, learning activities in the form of a series of sequential tasks, and hypotheses of the learning process that predict the flow of students' thinking development.

Hypothetical Learning Trajectories in Mathematics Learning. Hypothetical learning trajectory according to Clements & Sarama (2004) is a picture of students 'thinking during the learning process in the form of guesses and hypotheses from a series of learning designs to encourage the development of students' mathematical thinking so that learning objectives can be achieved as expected. In addition, Rezky (2019) explains hypothetical learning trajectory (HLT) which includes a series of instructional tasks in order to understand students' concepts of mathematics learning, is one of the important aspects that must be possessed by teachers in teaching students meaningful learning. This is because in HLT it is very considering the knowledge possessed by students. According to Fuadiah (2017), HLT is prepared by paying attention to the stages of the student's thought flow and the material concepts that students must build. Both of these must synergize with each other so that the activities designed are in line with both so that a learning design is found that suits the learning needs and characteristics of students. Correspondingly, Suwarto and Purnami (2018), suggested that designing the learning process with HLT starts with finding student learning difficulties (learning obstacles) through assessment, then using information from the assessment to develop didactic designs that can help students achieve learning goals. This didactic design is outlined in the RPP. Therefore, lesson plans should be built on the things that are considered difficult for students. Thus, hypothetical learning trajectories in mathematics learning are the teacher's guesses on mathematics learning activities (interactions with students) arranged
based on students' initial knowledge, and analysis of the location of students' learning difficulties (learning obstacle) to build students' mathematical thinking flow towards learning objectives.

In addition, an example of the application of hypothetical learning trajectory in other mathematics learning is an example of a hypothetical learning flow for learning the concept of place values by Novita & Putra (2017). The resulting learning flow can be seen in Figure 1.

In the compiled learning trajectory, learning begins with a contextual problem which is to help the ice cream maker in counting the number of remaining ice cream sticks. After that, the teacher tries to direct students to use the ten grouping technique by comparing which is the easier way to calculate one by one or by grouping them into ten-ten. The terms "tie stick" and "stick bag" are introduced by the teacher as a non-proportional model with the aim of directing students to find the concept of place values of hundreds, tens and units. In addition, the process of grouping and trading rules is also illustrated by the teacher through animated video as an activity to summarize the activities that have been carried out by students.

**Empirical Results**

Results of research conducted by Zuliana (2017) in her article entitled "Desain Siputmatika dan Rancangan Lintasan Belajar Siswa Sekolah Dasar Pada Materi Simetri Putar" produces a learning trajectory design (learning trajectory) of rotary symmetry using HLT arranged according to the difficulty of learning mathematics that occurs when students dealing with formal mathematical concepts related to the symmetry of a flat figure. Students still memorize concepts in learning the axis of symmetry, the amount of folding symmetry and the rotational symmetry of a flat figure. As a result, many students experience errors and are reversed by the concept of symmetry, advanced problems occur when students are asked to prove the position of the symmetry axis, the amount of folding symmetry and the amount of rotational symmetry of a flat figure. This condition shows the gap between abstract mathematical study objects about the concept of symmetry of flat shape and the level of cognitive thinking of elementary school students who are more interested in concrete objects.
Therefore used Siputmatika as a learning medium. In terms of the content and concept of the Siputmatika media, it is equipped with a shaft or center point as the center to rotate, the rotation angle is used to help students see the extent to which the media is rotated, the flat frame is used as a reference. This flat frame has a shape adjusting the shape of the flat shape which will be determined by the amount of rotational symmetry. Next in this siputmatika there is also a bag of rules that are located on the left. This bag is used as a guide to using the siputmatika visual aid. The right pocket is a flat bag that contains various types of flat plates.

**Figure 2. Model Alat Peraga Siputmatika**

The HLT is designed as presented in Figure 3.

**Figure 3. HLT pada materi Simetri Putar Bangun Datar menggunakan Siputmatika**

Learning trajectory design: hypothetical learning trajectory (HLT) material for Symmetry Play Flat Build for elementary school students is further described in Table 1 below.
Table 1. HLT Simetri Putar Bangun Datar pada Siswa Sekolah Dasar Menggunakan Siputmatika

<table>
<thead>
<tr>
<th>No</th>
<th>Tujuan Pembelajaran</th>
<th>Kegiatan Pembelajaran</th>
<th>Dugaan/hipotesis</th>
<th>Konsep</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Siswa mampu mengingat kembali jenis dan bentuk bangun datar</td>
<td>Melalui pengamatan, siswa mengamati beberapa bangun datar untuk kemudian mencari bingkai bangun datar yang sesuai</td>
<td>Siswa mampu menyebutkan jenis dan bentuk bangun datar serta menemukan dengan tepat bingkai bangun datar yang sesuai</td>
<td>Ketepatan jenis dan bentuk bangun datar: persegi, persegi panjang, segitiga, belah ketupat, jajar genjang, layang-layang, trapesium dan lingkaran</td>
</tr>
<tr>
<td>2</td>
<td>Siswa mampu menemukan konsep simetri putar</td>
<td>Melalui percobaan (eksperimen) siswa menggunakan siputmatika untuk mencari konsep simetri putar dari suatu bangun datar</td>
<td>Siswa mampu memutar dengan benar sesuai petunjuk penggunaan peraga siputmatika, untuk selanjutnya mendapatkan konsep simetri putar ketika bangun datar tersebut mampu menempati bingkainya dengan tepat</td>
<td>Konsep simetri putar</td>
</tr>
<tr>
<td>3</td>
<td>Siswa mampu menentukan jumlah simetri putar bangun datar</td>
<td>Melalui percobaan (eksperimen) siswa menggunakan siputmatika untuk menemukan banyaknya simetri putar bangun datar</td>
<td>Setelah mengetahui konsep simetri putar, siswa mampu melakukan percobaan menentukan jumlah/banyaknya simetri putar bangun datar</td>
<td>Banyaknya simetri putar bangun datar</td>
</tr>
</tbody>
</table>

Results of research conducted by Marion, Zulkardi, & Somakim (2015) in a journal entitled "Desain Pembelajaran Pola Bilangan Menggunakan Model Jaring Laba-laba di SMP". Based on preliminary research, some students have difficulty in understanding Number Patterns.
Especially in the case of mathematical modeling which is a process that begins from looking at real phenomena and efforts to mathematical phenomena. Only four to six students who can be said to be sufficiently understand the pattern of numbers based on students' strategies to solve problems about number patterns. The rest did not understand, there was even a tendency to memorize the formulas in the book. The use of the Spider Web Model in learning number patterns with woven craft contexts can help students understand number patterns with the learning trajectory as follows: (1) Recognize woven motifs and describe one pattern from woven motifs and continue the next patterns regularly. (2) Finding the sequence of numbers as a representation of the shape of each shape in a sequence. (3) Finding a recursive relationship on the sequence of numbers, i.e., continuing the next terms of the sequence of numbers. (4) Finding rows of numbers as representations of many square units of each pattern image of woven motifs. (5) Finding the functional relationship of the nth term of a sequence of numbers. (6) Using the nth term functional relations of rows of numbers in problem solving. (7) Through designing their own patterns for woven motifs related to the sequence of numbers given. (8) Determine the sum up to n the first term of a simple series. (9) Designing woven motifs based on the series given as an effort to foster an entrepreneurial spirit.

**Figure 4. HLT Pada Materi Pola Bilangan**

Results of research conducted by Hermanto & Santika (2017) in his article entitled "Eksplorasi Epistemological dan Didactical Obstacle serta Hypothetical Learning Trajectory Pada Pembelajaran Konsep Jarak" based on the observations of researchers of class X students in one of the high schools in Tasikmalaya City consisting of 32 students, found some student difficulties regarding the concept of distance in the building space. The most common difficulty among them is the difficulty of students in determining the projection of points against lines or planes. Students also do not master the concept of a point projection correctly on the plane, resulting in geometrical concept errors. Thus, it needs to be explored in more depth, what are the things that cause the weak mastery of students' geometrical concepts, especially regarding the concept of point-to-line projections and point-to-plane projections. The researcher predicts more broadly, it is likely that students will have difficulty projecting...
lines in the plane, because to project lines in the field involves more complex geometrical concepts. Besides that, the visualization ability of students in seeing the whole building has not been really well mastered. This greatly affects students when solving problems regarding the concept of distance in the building space, especially when students transform the flat figure depicted in the third dimension into the second dimension.

Based on the study of students’ obstacle learning on the material distance between points and lines, the researcher arranged the hypothetical learning trajectory (HLT) as follows.

**Figure 5. HLT konsep jarak dari titik ke garis**

Figure 5 shows that after students understand the concept of distance from points to lines through the problems that are around students, students are directed to understand the concept of distance from points to lines. Furthermore, students are trained to find the projection point through observation using the Cabri 3D software. At this stage the use of Cabri 3D software is very necessary to accelerate the process of action in learning. In addition, by using the 3D Cabri software students can conduct investigations to identify the shape of the field to be formed (making a plane). Then students can enter the final stage of calculating the distance from point to line by using various concepts according to the type of field formed. Hypothetical Learning Trajectory (HLT) material learning distance between points and fields, is presented in Figure 3.6 below.

**Figure 6. HLT materi jarak titik ke bidang**

Based on Figure 6 above, researchers compiled HLT learning material from point to field distances. Not much different from learning distance from point to line material, in this material learning begins by introducing students to problems related to the concept of distance from points to fields. Furthermore, by using the 3D Cabri software, students are
directed to investigate and make observations in finding line segments in the plane (finding a segment) which contains a projection of a point in that field (projection). After that, students are directed to the next stage which is to make a field that contains points and line segments found in the previous stage, and ends by calculating the distance from points to fields using various appropriate concepts.

Research conducted by Zuliana (2017) produced a hypothetical learning trajectory design using Siputmatika (Mathematical Play Symmetry). Hypothetical learning is designed here based on Siputmatika learning media as explained in the empirical results section. The use of this teaching aid can make it easier for students to understand concepts with direct experience when conducting experiments. This model of teaching aids can also be imitated for use by teachers in the learning of the symmetry of the play flat build. However, of course the learning time needed will be longer, because students must try one by one during the experimental process, plus the characteristics of elementary school students with a high curiosity in conducting experiments with teaching aids. Teachers also need more time and money to make Siputmatika teaching aids in large quantities.

Research conducted by Marion, Zulkardi, & Somakim (2015) produces learning trajectories that can help students understand Number Patterns in the context of woven crafts as well as giving students the opportunity to build their own knowledge using the Spider Web learning model in junior high. The woven cultural context can be used as a starting point in learning mathematics, because woven motifs show regular and even repetitive patterns, so that it contains mathematics. HLT designed can be applied in learning in schools. Teachers in NTT can adjust the webbing to be used with woven motifs in the NTT area. Furthermore, the existing HLT can be further developed as needed.

For example, from the motif of Timor woven fabric and Rote woven fabric, it is seen that the shape follows the pattern of numbers as follows.

![Figure 7. Pola bilangan dalam kain tenun khas Timor](image)

![Figure 8. Pola bilangan dalam kain tenun khas Rote](image)

The used of woven fabric motifs in learning number patterns can support students to construct their own thinking by finding varied patterns.
Research conducted by Hermanto & Santika (2017) compiled hypothetical learning trajectory (HLT) based on a study of student obstacle learning on the material distance between points and lines. HLT which was designed also uses the help of Cabri 3D geometry software that makes it easy for students and teachers to explore various geometric shapes and models. This HLT design is very good to be applied in schools as an innovation in IT-based learning. In addition, the structured learning process starts from understanding the concept of distance from point to line through problems around students, students are directed to understand the concept of distance from point to line. Furthermore, students are trained to find the projection point through observation using the Cabri 3D software. At this stage the use of Cabri 3D software accelerates the process of action in learning. In addition, by using the 3D Cabri software students can conduct an investigation to identify the shape of the field that will be formed (making a plane). Then students can enter the final stage of calculating the distance from point to line by using various concepts according to the type of field formed. Therefore, this design is highly recommended for use, and can be developed.

CONCLUSION
Learning trajectory is a series of learning activities that are designed with more effective teaching strategies based on problems faced by students to encourage the development of student thinking to achieve learning goals. Meanwhile, hypothetical learning trajectory is a learning design in the form of allegations of student learning activities based on initial understanding and characteristics of students to achieve higher understanding. Hypothetical learning trajectory serves as a guide for teachers to predict and prepare learning flow designs that are appropriate to the stages of student thinking and can improve student learning outcomes. In general, hypothetical learning trajectory has three main components consisting of learning objectives, learning activities, and hypotheses of the learning process that predict the flow of students' thinking development.

In mathematics learning, hypothetical learning trajectories are designed with regard to student obstacle learning, student's initial knowledge, and learning material, so that learning activities that will be hypothesized for students to achieve learning objectives are built based on these things. Many hypothetical learning trajectories have been designed and applied in mathematics learning, including HLT design in learning on material Symmetry Play (Zuliana, 2017), Number Patterns (Marion, Zulkardi, & Somakim, 2015) Distance between points and lines using the help of geometry software Cabri 3D (Hermanto & Santika, 2017).

REFERENCES


Agnes et al: Study of Hypothetical Learning ... 79

mathematics education (pp. 429-466). Springer, Dordrecht. Doi: 10.1007/978-94-017-9181-6_16

*Mathematical Thinking And Learning, 6*(2), 81-89.

learner-centered curricular coherence using learning maps and diagnostic assessments 
designed around mathematics learning trajectories. *ZDM Mathematics Education.* 
Springer. Doi: 10.1007/s11858-017-0869-1

Dalziel, J. (2015). *The Art & Science of Learning Design.* In: Maina, M., Craft, B., and Mor, 

growth learning trajectory: students’ emerging understanding of exponential growth 


hypothetical learning trajectory pada pembelajaran konsep jarak. *Jurnal Penelitian 
Pendidikan dan Pengajaran Matematika, 3*(2), 115-128.


conceptualizing matrices as linear transformations. *International Journal of 
Mathematical Education in Science and Technology, 48*(6), 809-829.

Marion, Zulkardi, & Somakim. (2015). Desain pembelajaran Pola Bilangan menggunakan 
model Jaring Laba-laba di SMP. *Jurnal Kependidikan, 45*(1), 44-61.

berbantuan video animasi terhadap pemahaman konsep nilai tempat siswa kelas II SD. 
*Jurnal Pendidikan Matematika, 11*(1), 43-56.

education to improve the mathematical communication of junior high school students. 
*Journal of Mathematics Education. 8*(2), 247-258.


